AN ALTERNATIVE METHOD FOR YOUNG'S MODULUS DETERMINATION BY RESONANT FREQUENCY FOR GENERAL SAMPLE GEOMETRIES

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INTRODUCTION

At RHI the resonant frequency method is used for elastic material properties determination of refractory samples with different geometric scales. When the resonant frequency method is used as a pre-testing tool for round robin tests [1] many samples with different geometries must be tested. First of all the sample scale is large for refractory bricks, after shaping the geometry size is small for laboratory samples. The Young's Modulus determination by the resonant frequency method described [2, 3, 5] is time and money consuming, because the measurement and analysis procedure contains many steps. This time must be multiplied by the number of samples and necessary measurement repetitions. Therefore, a new method was researched to reduce the time demand for the resonant frequency based Young's Modulus determination. In this paper a solution is described which reduces the measurement steps of the standardized Young's Modulus determination method. The new Young's Modulus determination procedure is verified in this paper by a number of investigations.

FUNDAMENTALS

The Resonant Frequency Method

As resonant frequency determination method the impulse excitation technique (IET) [1] was used. The IET-method is a nondestructive testing method which determines the frequency spectrum of a sample with a mechanical hammer impulse which is excited at the specimen surface. The mechanical excitation causes a vibration of the specimen.

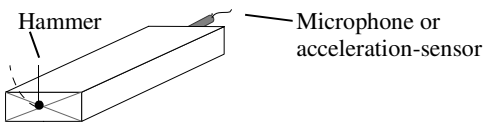


Fig. 1: Longitudinal wave-type IET-measurement setup [2]

For this investigation the excitation is defined for two different wave types, the longitudinal (Fig. 1) and the flexural waveform (Fig. 2). The flexural waveform is measured in two orientations of the rectangular cross sectional area of a specimen. The directions named flexural in-plane and flexural out-of-plane [3] (Fig. 2). A waveform is also named vibration-mode in this paper.

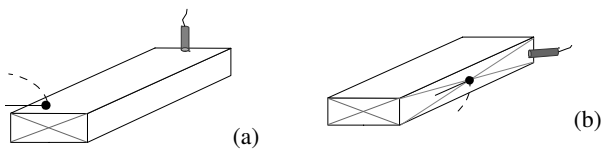


Figure 2: The flexural wave-type IET-measurement setup with the two orientations, (a) the flexural out-of-plane and (b) the flexural in-plane excitation direction [3]

The vibration is measured by a microphone as a time dependent signal. This signal is analyzed by the fast Fourier transform (FFT) analysis, which produces a frequency spectrum.

Numerical Sequence of Frequencies

The numerical sequence of frequencies $f_{(i)}$ is used for the alternative Young's Modulus determination procedure. The numerical sequence of frequencies is described by equation (1).

$$f_{(i)} := \begin{cases} f_{(i)} > 0 \\ f_{(i)} < f_{(i+1)} \end{cases} \; for\ i = 1, \dots, n \tag{1}$$

A FFT analysis of a signal causes a frequency spectrum (Fig. 3). In this spectrum the frequency values with the highest amplitudes are determined; they are called main-frequencies. This procedure is done for all wave types measured with the resonant frequency determination method. All detected main frequencies collected in a numerical sequence of frequencies.

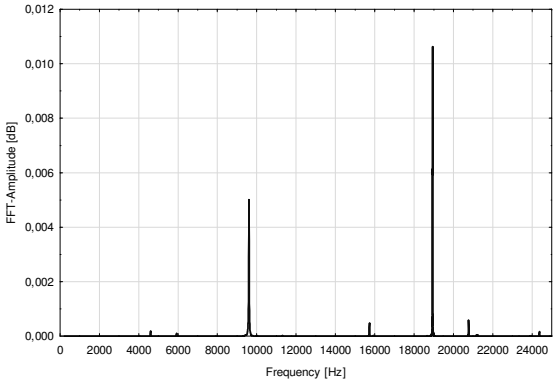


Fig. 3: Frequency spectrum example with two high amplitude frequencies at 9.6 kHz and 18.9 kHz

Young's Modulus formulas

The computation of the Young's Modulus (YM) or elastic modulus E is described here. In equation (2) the calculation of the longitudinal Young's Modulus E_l [1, 4, 5] with the frequency f_l is shown.

$$E_l = \left(\frac{f_l}{n}\right)^2 \rho l^2 4T \tag{2}$$

l is the length, n is the number of the harmonics of the resonant frequency f_l, ρ is the density and T is a geometry depended parameter.

The flexural elastic modulus E_f [5] of the flexural frequency f_f is computed with equation (3).

$$E_f = C\,\frac{4\pi^2 l^3 {f_f}^2 m}{\lambda^4 I} \tag{3}$$

m is the mass of the specimen, I the second moment of area and C is a parameter which depends from the specimen dimension, the Poisson's ratio μ.

$$\lambda = \frac{1}{2}(2n-1)\pi \tag{4}$$

λ is a parameter which describes the number of harmonics n of the frequency f_f. The flexural waveform can be measured in two orientations of the specimen (Fig. 3). Therefore the flexural in-plane resonant frequency is named f_{fip} and the flexural out-of-plane resonant frequency is named f_{fop} [3]. With the in-plane and the out-of-plane specimen orientation it is necessary to consider the orientation of the cross sectional area in the calculation of the parameter C. In this investigation geometries with a rectangular cross-section are used, but it is also possible to investigate samples with circular or quadratic cross-section area.

Resonant frequency computation

The longitudinal resonant frequency f_l is solved by equation (2) and shown in equation (5).

$$f_l = \frac{n}{2l}\sqrt{\frac{E_l}{\rho T}} \tag{5}$$

Formula (6) shows the calculation of the flexural resonant frequency f_f solved for equation (3).

$$f_f = \frac{\lambda^2}{2\pi}\sqrt{\frac{E_f I}{C l^3 m}} \tag{6}$$

A specimen dimension with a ratio of the length to the minimum rectangular cross-sectional area dimension greater than one have a descending order of the resonant frequencies, see equation (7).

$$f_l > f_{fip} > f_{fop} \tag{7}$$

Resonant frequency relations and ratio calculation

Resonant frequency relations depend primarily on the specimen geometries and then on the Poisson's ratio of the investigated material. This information is necessary when a plausibility check of the Young's Modulus results is carried out. The resonant frequency ratios r_1, r_2 and r_3 are computed with the equations (8) to (10).

$$r_1 = \frac{f_l}{f_{fip}} \tag{8}$$

$$r_2 = \frac{f_l}{f_{fop}} \tag{9}$$

$$r_3 = \frac{f_{fip}}{f_{fop}} \tag{10}$$

The resonant frequency ratios r_1, r_2 and r_3 depend on the specimen dimension and Poisson's ratio and are independent of the Young's Modulus [6]. The frequency ratio values can be influenced by the anisotropy of the investigated refractory material [7].

Finite Element Simulation

The finite element simulation (FEM) was done by software Abaqus [8]. It was used to obtain the Eigenfrequency value and a deformation image of a sample. For the FEM simulation the specimen dimension, the Young's Modulus and the Poisson's ratio are needed.

In the finite element Eigenfrequency analysis a special type of equation of motion [9] of an elastic solid object is used, see equation (11).

$$([K] - \omega^2[M])\{u\} = \{0\} \tag{11}$$

[M] is the mass matrix, ω the circular frequency, [K] is the stiffness matrix and {u} is the displacement vector. With the circular frequency $\omega = 2\pi f$ the eigenfrequency f can be calculated.

For a simulation task equation (11) is applied with an adapted solving algorithm [9] of the FEM software [8]. The solving algorithm produces results with a determination error which causes an unsharpness of the Eigenfrequency results. The deformation image of each Eigenfrequency result is used to identify the vibration mode of the Eigenfrequency result.

Ultrasonic Transmission Method

The ultrasonic transmission method (UT) [4] was applied to determine the dynamic Young's Modulus of the specimen. According to the investigation procedure which is described in [4] a number of measurement points at the sample surface were measured and velocities calculated. The average value of the velocities v was computed and then the Young's Modulus E_{ut} of the sample was calculated using equation (12).

$$E_{ut} = v^2 \rho \frac{(1+\mu)(1-2\mu)}{1-\mu} \tag{12}$$

THE ALTERNATIVE YOUNG'S MODULUS DETERMINATION METHOD

The alternative method for Young's Modulus determination (AYMD) is an algorithm which performs all determination steps, see [1 - 6], automatically. It searches, calculates the Young's Modulus values from a numerical sequence of frequencies and the result is judged by the AYMD algorithm. The following input data is required:

- A numerical sequence of frequencies which is a summary of all main-frequencies of the FFT-spectra determined by the longitudinal, the flexural in-plane and the flexural out-of-plane waveform
- The specimen dimension and the density value, a hypothetical value of the Poisson's ratio
- The expected Young's Modulus value with a deviation range for the plausibility-check
- A deviation-range of the frequency relations r_1, r_2 and r_3, e.g. ±1%

A flowchart of the AYMD-procedure is shown in Fig. 3. At first the AYMD-algorithm computes the frequency relations computed with a variation of the Poisson's ratio, an example is shown in Table 1.

Tab. 1: Frequency relations for Sample A with Poisson's ratio variation

Poisson's ratio	r1	r2	r3
0	1.607	2.025	1.260
0.15	1.653	2.062	1.247
0.3	1.699	2.099	1.235

Then a heuristic search algorithm looks for frequency values which fulfill the conditions of the frequency relation values r_1, r_2 and r_3. By the relations also the wavetypes are expected. The quality of the search result depends on the number of relations accordance's found. If no frequency relations are found the procedure is stopped.

If the frequency relations are correct the calculation of the Young's Modulus values E_l, E_{fip} and E_{fop} is done. A plausibility test of the Young's Modulus results is the final step of the AYMD-procedure.

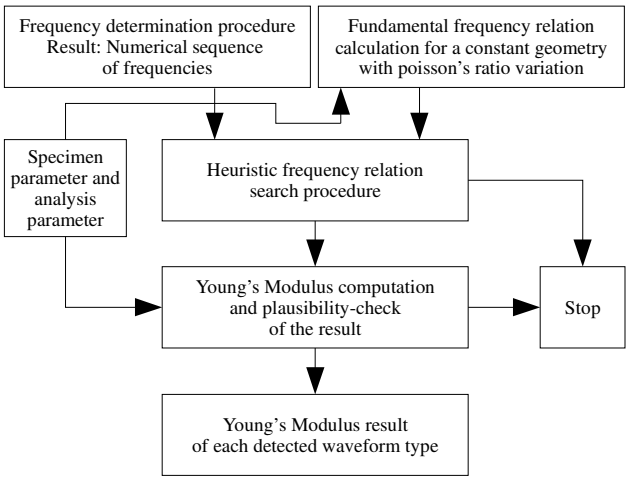


Fig. 4: Flowchart of the alternative method for Young's Modulus determination

The AYMD algorithm provides the results as Young's Modulus values with the wave type and the frequency value. All steps of the alternative method for Young's Modulus determination can be realized by any programming language.

EXPERIMENTAL

In the experimental section of this work the results of the AYMD procedure was verified by several investigations. It contains FEM Eigenfrequency simulations of samples with different geometries, Young's Modulus values, densities, and Poisson's ratios. For refractory samples the resonant frequency method and the ultrasonic transmission method were applied. The AYMD algorithm was verified by the Young's Modulus value and type and by the accordance of the frequency relation values. All measurements and FEM simulations carried out by the Technical Center Leoben of the RHI-AG.

Samples and investigation parameter

In Tab. 2 the sample geometries are shown.

Tab. 2: Sample geometries

Geometry No	Length [mm]	Width [mm]	Thickness [mm]
#1	266	116	77
#2	266	77	55
#3	266	57	55

The specimen dimension and material data for the FEM-Eigenfrequency simulation is described in Tab. 3.

Tab. 3: Input data for the FEM simulations

Sample No	Dimension No	YM [GPa]	Density [g/cm3]	Poisson's ratio
A	#1	86	3.3	0.15
B	#2	86	3.3	0.15
C	#3	86	3.3	0.15

The chemical composition of the investigated refractory samples is shown in Tab. 4.

Tab. 4: Chemical composition of the refractory material

MgO [wt-%]	Cr_2O_3 [wt-%]	Al_2O_3 [wt-%]	Fe_2O_3 [wt-%]	Density [g/cm^3]
56	21	7	13	3.3

In Tab. 5 the investigation methods are described which were applied at refractory samples.

The ultrasonic transmission method was used for verification of the Young's Modulus values which are computed by the AYMD algorithm out of the IET-measured frequency spectra, see Tab. 5.

Tab. 5: Investigated refractory samples with the chemical composition shown in Tab. 4 by the resonant frequency method IET and ultrasonic method UT

Sample No	Geometry No	Measurement method
D	#1	UT, IET
E	#2	UT, IET
F	#3	UT, IET

Finite-Element Simulation

The finite element simulation results a numerical sequence of Eigenfrequencies and deformation images. The deformation images are used to identify the wavetype and to identify errors of the FEM solving algorithm [9]. In Fig. 5 an overview of FEM Eigenfrequencies of the FEM simulation of the samples A, B and C is shown. The FEM simulation results in more than 1000 Eigenfrequencies per sample, the number of results depends on the chosen boundary condition of the simulation.

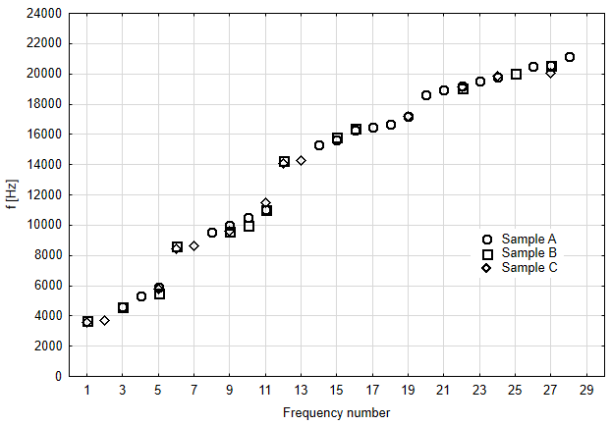
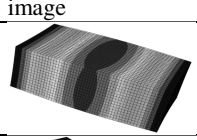
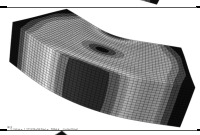
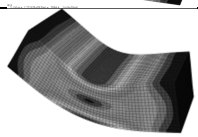
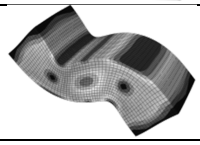


Fig. 5: Overview of the first 24 Eigenfrequencies computed by FEM of the samples A, B and C

RESULTS

Initially the AYMD algorithm was applied at the Eigenfrequency FEM simulation results of sample A, B and C which are shown in Fig. 5.

Tab. 6: AYMD Young's Modulus determination of the FEM-simulation of sample A, verified with the FEM deformation images

E [GPa]	f [Hz]	Waveform-type	FEM deformation image
85.5	9566	Longitudinal 1st harmonics	
85,4	5894	Flexion in-plane 1st harmonics	
85.5	4608	Flexion out-of-plane 1st harmonics	
85.9	9975	Flexion out-of-plane 2nd harmonics	

The results were verified with the deformation images of each Eigenfrequency which was found by the AYMD algorithm. Tab. 6 shows the verification for sample A, the result was successful, each determined Young's Modulus type fits with the deformation image of the found frequency value.
In Tab. 7 the results of the application of the AYMD algorithm for the samples A, B and C are shown. The application was also successful because all Young's Modulus types and their frequencies fit the FEM deformation images. The calculated Young's Modulus values are lower than the input value of the FEM simulation (Tab. 3). This was the result of the FEM solving algorithm which stops to early. Therefore the Young's Modulus values calculated with the FEM-Eigenfrequency results are always lower than the FEM input Young's Modulus value.

Tab. 7: Young's Modulus determination results of the AYMD algorithm at FEM Eigenfrequencies

Sample No	E_l [GPa]	f_l [Hz]	$E_{f\text{-}op}$ [GPa]	$f_{f\text{-}op}$ [Hz]	$E_{f\text{-}ip}$ [GPa]	$f_{f\text{-}ip}$ [Hz]
A	85.5	9566	85.5	4608	85.4	5894
B	85.8	9582	84.1	3693	85.2	4608
C	85.8	9586	84.2	3590	85.0	3692

The AYMD algorithm was also verified with refractory samples. As verification the ultrasonic method was used. The frequency spectra were determined by the resonant frequency method (IET). The results are shown in Tab. 8.

Tab. 8: Young's Modulus verification of refractory samples. μ* was estimated by the AYMD algorithm

	Ultrasonic method		AYMD algorithm (IET)			
Sample No	E_{ut} [GPa]	μ [1]	E_l [GPa]	$E_{f\text{-}op}$ [GPa]	$E_{f\text{-}ip}$ [GPa]	μ* [1]
D	81.4	0.15	86.2	86.2	86.9	0.2
E	81.9	0.15	88.2	86.0	86.0	0.1
F	84.6	0.15	87.5	87.0	---	0.3

The deviations between the ultrasonic and the IET method are between three and five percent. The AYMD algorithm works well, with only sample F, the frequency of flexural-in-plane wave-type could not be determined. With the AYMD algorithm it is also possible to estimate the Poisson's ratio, this is shown in Tab. 8.

Tab. 9: Contrasting juxtaposition of frequency ratios of FEM simulation data and measurement data determined by the AYMD algorithm for equal specimen geometries

Geometry No	Sample No	Frequency ratios [1]		
		r_1	r_2	r_3
#1	A	1.622	2.088	1.287
	D	1.623	2.076	1.279
#2	B	2.095	2.593	1.238
	E	2.079	2.595	1.248
#3	C	-	2.607	-
	F	2.596	2.670	1.028

Tab. 9 shows the comparison of frequency ratios computed by the AYMD algorithm. The raw is provided by the FEM simulation (sample A, B, C) and from resonant frequency measurement for refractory samples (sample D, E, F). For identical sample geometries the results show a good accordance between determined frequency ratios.

CONCLUSION

The alternative Young's Modulus determination method uses the principle of free vibration of all wave types and orders in specimen geometry. This alternative resonant frequency Young's Modulus determination method was verified in several ways. It was tested on different sample geometries and refractory material types. For a methodical verification the standard resonant frequency method according to ASTM C1259-08 and the ultrasonic transmission method were used to determine the Young's Modulus values. For a theoretical verification, the finite element Eigenfrequency calculation was applied for different geometries and material properties. In the Eigenfrequency data the alternative Young's Modulus determination method was applied and the results compared with the finite element simulation parameters of the data set. All investigations were carried out in the Technical Center Leoben of the RHI-AG.

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